Supplementary Appendix to Trade and Sectoral Productivity

Harald Fadinger and Pablo Fleiss

1. Robustness

In this section we perform several robustness checks on our productivity estimations. We try alternative econometric specifications and then discuss the effects of changing particular assumptions of our model. Moreover, we compare our productivities with those computed as Solow residuals for the few countries and sectors where this measure can be constructed.

1.1. Heterogeneous Firms and Zeros in Bilateral Trade

Up till now we have assumed that firms are homogeneous and that there are no fixed costs to export, so that all firms in a given sector k of country j are predicted to export to every country i. In reality, only a fraction of firms export and very few firms export to several destinations. In addition, we have ignored zeros in bilateral trade flows, which are quite prevalent in the data[1], hence our estimates are conditioned on observing positive trade flows. Helpman et al. (2008) argue that one needs to take these facts into account in order to obtain unbiased estimates for the impact of distance and other bilateral variables on trade flows when modeling the volume of bilateral trade with gravity type regressions. Firm heterogeneity matters because the number of firms engaged in bilateral trade (the extensive margin) varies systematically with trade costs. Only the most productive firms can sell enough to recoup the fixed costs to export to destinations with high

[1] In the mid-90's 8907 out of 51029 possible trade flows are zero in our sample.

marginal trade costs. Not to consider the extensive margin means to confuse the impact of trade barriers on the number of firms with the effect on exports per firm and this leads to biased estimates of the latter effect.

Zeros in bilateral trade matter because of sample selection. Observing positive trade flows is not random because many of the variables that determine bilateral fixed costs to trade – and therefore firms' decision whether to export or not – also affect the variable cost to trade and thus our measure of "raw" productivity. Country-sector pairs with large observed barriers that trade a lot are likely to have low unobserved trade barriers, which may violate our assumption that the unobserved variation of raw productivity across importers for a given exporter-sector is not systematic.

In this section we check if our productivity estimates are robust to controlling for these factors. We follow the approach suggested by Helpman et al. (2008), which forces us to use a somewhat different specification for our productivity estimates and obliges us to use information on the number of firms active in the exporting country, which we consider less reliable than the data on aggregate production. Nevertheless, our results on productivities remain quite similar. Since the derivation of the estimating equations of this extension requires a fair amount of additional algebra, we just present the final specification.[2]

We assume that the inverse of firm's productivity is drawn from a distribution with cumulative distribution function $G_{jk}(a) = 1/A_{jk}G(a)$ with support on $[a_{Lk}, a_{Hk}]$, that can be written as the product of a country-sector specific term and a distribution that is invariant across countries. One can show that A_{jk}, which can be interpreted as an average of the sectoral efficiency level in the exporting country and that we refer to as sectoral productivity,[3] can be recovered from the following expression.

$$E[\log(\tilde{A}_{ijk})|X_{ijk}, T_{ijk} = 1] = \log(A_{jk}) + D_{ik} + \beta_k X_{ijk} + \frac{1}{\epsilon_k - 1} E[\log(V_{ijk})|T_{ijk} = 1] + E[e_{ijk}|T_{ijk} = 1], \quad (1)$$

where $E[\log(\tilde{A}_{ijk})|.]$ is the mathematical expectation of raw productivity conditional on a vector of bilateral variables X_{ijk} and on observing positive trade flows, $T_{ijk} = 1$. The term $E[\log(V_{ijk})|T_{ijk} = 1]$ controls for

[2]Derivations can be found in the working paper version of this paper (Fadinger and Fleiss (2008))

[3]A more standard definition of sectoral productivity would be $\check{A}_{jk} \equiv A_{jk} \left(\int_{a_{Lk}}^{a_{jk}} a^{1-\epsilon_k} dG(a) \right)^{\frac{1}{1-\epsilon_k}}$, a weighted mean of firm productivities. The cutoff a_{jk} is endogenous and depends on the level of competition in the exporting country. See Melitz (2003). Our definition disregards the effect of firm selection on the level of sectoral productivity.

the fraction and the productivity composition of exporters from country j that export to country i in sector k and $E[e_{ijk}|T_{ijk} = 1]$ controls for the sample selection because of unobservable trade barriers that affects both the decision to export and the volume of trade, while D_{ik} is a importer-sector dummy. In the working paper version of this paper we derive a consistent estimator for this conditional expectation that can be implemented with a two-step selection model. For each destination firms first choose whether to export or not and if so how much to export.

Table 1 shows the correlations and rank correlations between our baseline productivity estimates and different specifications. The second specification considered ignores the issues of sample selection and heterogeneous firms to check how much results are affected by using the number of firms instead of aggregate production in our productivity estimations (columns labeled "number of firms"). We can see that the results are quite similar except for the sectors of Pottery and Scientific Equipment. In the next columns we take care of the issue of zero trade flows by estimating a standard Heckman-selection model (columns labeled "Heckman"). The inverse Mill's ratio enters positively and significantly in all sectors, so that there is indeed sample selection towards countries with low unobserved trade barriers.[4] However, results for productivities change very little compared to the specification that only uses the number of firms. Finally, we simultaneously control for sample selection and the extensive margin of trade (via a 3rd order polynomial approximation of $E[log(V_{ijk})|T_{ijk} = 1]$) – columns labeled "heterogeneous firms". Even though these terms are all significant,[5] correlations and rank correlations for our productivities remain around 0.8, so that our baseline specification seems to be robust.

1.2. Eaton and Kortum's (2002) Model

An alternative model for estimating sectoral productivities from trade data is Eaton and Kortum (2002)'s model of trade. This is a Ricardian model that can easily be extended to the Heckscher-Ohlin style trade. Chor (2010) uses a version of this model to divide comparative advantage into different components by proxying for technology differences with observables but he is not specifically interested in measuring sectoral

[4] Results not reported but available on request.
[5] Results not reported but available on request.

TFPs. Finicelli et al. (2008) apply the baseline Eaton-Kortum model to calibrate aggregate manufacturing TFPs for a number of OECD economies, focusing on the role of competition on TFP, which we disregard in our discussion. While we define productivity as the average technology level, they focus on the effect of trade openness on the firm composition and hence on the aggregate productivity.

The model assumes a fixed measure of varieties $n \in [0,1]$ in each sector and perfect competition so that firms price at their (constant) marginal cost and countries source a given variety exclusively from the lowest cost supplier. The price of variety n of sector k produced in country j as perceived by country i consumers is

$$\hat{p}_{ijk}(n) = \frac{1}{A_{jk}(n)} \prod_{f \in F} \left(\frac{w_{fj}}{\alpha_{fk}} \right)^{\alpha_{fk}} \tau_{ijk}. \tag{2}$$

Here, $A_{jk}(n)$ is stochastic and parameterized such that $log(A_{jk}(n)) = \lambda_{jk} + \beta_k \epsilon_{ik}(n)$, where $\epsilon_{ik}(n)$ follows a Type I extreme value distribution with spread parameter β_k. This distribution has a mode of λ_{jk} and $E[log(A_{jk})] = \lambda_{jk} + \beta_k \gamma$, where γ is a constant.

Using the assumption of perfect competition and the properties of the extreme value distribution it can be shown that exports of country j to country i in sector k as a fraction of $i's$ sectoral absorption are given by Π_{ijk}, the probability that country j is the lowest cost supplier of a variety n to country i in sector k.[6]

$$\frac{M_{ijk}}{\sum_{j \in J} M_{ijk}} = \Pi_{ijk} = \frac{\left[\prod_{f \in F} \left(\frac{w_{fj}}{\alpha_{fk}} \right)^{\alpha_{fk}} \tau_{ijk} \right]^{-1/\beta_k} exp(1/\beta_k \lambda_{jk})}{\sum_{j \in J} \left[\prod_{f \in F} \left(\frac{w_{fj}}{\alpha_{fk}} \right)^{\alpha_{fk}} \tau_{ijk} \right]^{-1/\beta_k} exp(1/\beta_k \lambda_{jk})} \tag{3}$$

Consequently, normalizing with imports from the US,

$$\frac{M_{ijk}}{M_{iUSk}} = \frac{\Pi_{ijk}}{\Pi_{iUSk}} = \frac{\left[\prod_{f \in F} \left(\frac{w_{fj}}{\alpha_{fk}} \right)^{\alpha_{fk}} \tau_{ijk} \right]^{-1/\beta_k} exp(1/\beta_k \lambda_{jk})}{\left[\prod_{f \in F} \left(\frac{w_{fUS}}{\alpha_{fk}} \right)^{\alpha_{fk}} \tau_{iUSk} \right]^{-1/\beta_k} exp(1/\beta_k \lambda_{USk})}. \tag{4}$$

[6] For the derivations, see Eaton and Kortum (2002) or Chor (2010).

Taking logs, we get

$$\log\left(\frac{M_{ijk}}{M_{iUSk}}\right) = 1/\beta_k\left(\lambda_{jk} - \lambda_{USk}\right) - 1/\beta_k \sum_{f\in F} \alpha_{fk} \log\left(\frac{w_{fj}}{w_{fUS}}\right) - 1/\beta_k \log\left(\frac{\tau_{ijk}}{\tau_{iUSk}}\right). \qquad (5)$$

Thus, we obtain $E\left[\frac{log(A_{jk})}{log(A_{USk})}\right] = \lambda_{jk} - \lambda_{USk}$.[7]

The main difference between this specification and our model is that it requires no information on exporters' production. Relative exports depend exclusively on the relative probabilities of offering varieties in the importing market at the lowest cost, which depends only on bilateral variables, factor prices, and productivity.

To obtain productivity estimates from this model, we can either calibrate it by using information on the spread parameter β_k from other studies, or estimate it using our two step procedure.

When trying to estimate the equations with a two stage procedure analogous to the robustness check presented in the main text, many of the coefficients of relative factor prices have the wrong sign, so this specification seems to be performing poorly. Alternatively, we can apply the hybrid calibration and estimation exercise by first constructing raw productivities and then regressing these on bilateral variables. In order to do so, we require estimates of β_k. Chor reports an aggregate value of β of around 12.41^{-1}, Eaton and Kortum estimate β to lie between 2.44^{-1} and 12.86^{-1}. While the relative order of countries is meaningful for any β, the absolute size of productivity differences is very sensitive to the choice of β. Choosing a β of 12.41^{-1} (Chor's estimate) gives productivity estimates that are very similar to the ones obtained with our baseline model,[8] as can be seen in Table 1, where we report correlations and rank correlations by sector. When setting β equal to 2.44^{-1}, absolute productivity differences explode.

Hence, the Eaton-Kortum model seems to be a good alternative for estimating sectoral productivities. Its main advantage is that it does not require information on production, the drawback is that one has to estimate the spread parameter of the sectoral productivity distribution that is hard to pin down.

[7]Note that this is an estimate of the underlying technology parameter and not directly of realized TFP, which is the weighted average productivity of active firms only.

[8]The aggregate correlation is 0.89.

1.3. Pricing to the Market and Endogenous Markups

Mark-ups charged by exporting firms may depend on the level of competition in the destination market (Melitz and Ottaviano (2008), Sauré (2009)). In this subsection we study how our productivity estimation procedure is affected by the presence of pricing to the market. For doing so, we go back to our baseline model and slightly modify agents' utility function to make marginal utility bounded, so that consumers' demand drops to zero whenever a variety becomes too expensive.

$$u_{ik} = \left[\sum_{b \in B_{ik}} \ln(x_{bk} + 1) \right] \tag{6}$$

The demand for a sector k variety produced in country j by consumers in country i is now given by

$$x_{ijk} = \max\{ \frac{1}{\mu_{ik} \tau_{ijk} p_{ijk}} - 1, 0 \}, \tag{7}$$

where μ_{ik} is the shadow price of the sector k budget sub-constraint for country i consumers. Solving country j producers' profit maximization problem, one finds that exporters price discriminate across markets and set prices in destination i equal to a mark-up over their marginal cost that depends inversely on the toughness of competition in the export market, so that $p_{ijk} = \left(\frac{\frac{1}{A_{jk}} \prod_{f \in F} \left(\frac{w_{fj}}{\alpha_{fk}} \right)^{\alpha_{fk}}}{\mu_{ik} \tau_{ijk}} \right)^{1/2}$. Substituting into the definition of bilateral trade and simplifying we obtain

$$M_{ijk} = \mu_{ik}^{-1} \left\{ 1 - \left[\mu_{ik} \tau_{ijk} \frac{1}{A_{jk}} \prod_{f \in F} \left(\frac{w_{fj}}{\alpha_{fk}} \right)^{\alpha_{fk}} \right]^{1/2} \right\} N_{jk}, \tag{8}$$

whenever bilateral trade is positive.[9] Dividing by M_{iUSk}, taking logs and rearranging we get

$$\log \left(\frac{M_{ijk}}{M_{iUSk}} \right) - \log \left(\frac{N_{jk}}{N_{USk}} \right) \approx \left(\frac{A_{jk}}{A_{USk}} \prod_{f \in F} \left(\frac{w_{fUS}}{w_{jk}} \right)^{\alpha_{fk}} \frac{\tau_{iUSk}}{\tau_{ijk}} \right)^{1/2} \tag{9}$$

[9] Endogenous s are an alternative explanation to fixed cost to exporting for observing zeros in bilateral trade.

We see that the shadow price, μ_{ik} – which is the only term related to mark-ups and the level of competition in the export market – drops from the equation since exporters from country j and the US face the same level of competition in a given market i, but the relationship is no longer log linear. Moreover, N_{jk} cannot be replaced with aggregate production any more since the production level of individual firms q_{jk} depends on the trade weighted level of competition in the destination markets and prices charged in those markets, $N_{jk}\sum_{i\in I_{jk}} p_{ijk}q_{ijk}\tau_{ijk} = \tilde{Q}_{jk}$. Hence, our productivity estimation procedure remains approximately valid as long as we use the number of firms in the exporting country instead of aggregate production.

1.4. Trade in Intermediates

In this section we study how our specification is affected by the usage of tradable intermediate goods in production. Ethier (1982), Rivera-Batiz and Romer (1991), and others formalize the idea that having access to more varieties of differentiated intermediate goods through trade may boost sectoral productivity. Recently, Jones (2010) has emphasized that sectoral productivity may be crucially determined by linkages across sectors through the use of intermediate inputs, which may potentially lead to large multiplier effects of relatively small distortions. These ideas can easily be incorporated into our framework. We modify the production function in a way such that firms use not only capital and different labor types but also varieties of differentiated intermediates produced by other sectors (and potentially in other countries) as inputs. Assuming that firms spend a fixed fraction of their revenues on intermediates of each sector the cost function now becomes

$$TC(q_{ik}) = (f_{ik}+q_{ik})\frac{1}{A_{ik}}\left[\prod_{f\in F}\left(\frac{w_{fi}}{\alpha_{fk}}\right)^{\alpha_{fk}}\right]^{1-\beta_k}\left[\prod_{k'=1}^{K}\left(\sum_{b\in B_{ik'}}\hat{p}_{bk'}^{1-\epsilon_{k'}}\right)^{\frac{\sigma_{kk'}}{1-\epsilon_{k'}}}\right]^{\beta_k}, \tag{10}$$

where $\sum_{k'=1}^{K}\sigma_{kk'} = 1$ and $\epsilon_{k'} > 1$. Firms in sector k are assumed to spend a fraction, $\sigma_{kk'}\beta_k$, of their revenues on a CES aggregate of differentiated intermediate inputs produced by sector k' with elasticity of substitution $\epsilon_{k'}$.

Demand for intermediates by firms in sector k of country i for sector k' intermediates produced in country

j can be found applying Shepard's Lemma to (10),

$$x_{ijkk'} = \frac{\hat{p}_{ijk'}^{-\epsilon_{k'}} \sigma_{kk'} \beta_k N_{ik} TC(q_{ik})}{P_{ik}^{1-\epsilon_{k'}}}. \tag{11}$$

These demand functions can be easily aggregated over sectors k and combined with consumers' demand for varieties to get total bilateral demand for sector k' varieties. Hence, trade in intermediates does not change the value of imports from country j relative to those from the US, nor does it affect the functional form of our raw productivity measure relative to the US .

Since we do not explicitly take into account that firms use intermediates our measured productivity is $\breve{A}_{jk} \equiv A_{jk} \left[\prod_{k'} \left(\sum_{b \in B_{jk}} \hat{p}_b^{1-\epsilon_{k'}} \right)^{\frac{\sigma_{kk'}}{1-\epsilon_{k'}}} \right]^{-\beta_k}$. This implies that in countries and sectors where more varieties of intermediates are available and cheaper on average, measured productivity is higher. To the extent that intermediate inputs are non-tradable, like transport or government services, low productivity in other sectors leads to high prices of these intermediate inputs and consequently to lower measured sectoral productivity.

1.5. Mismeasurement of Sectoral Factor Income Shares

In our modeling procedure we have assumed that sectoral factor income shares do not vary across countries in order to be able to use the values of the US for these parameters, since reliable information on factor income shares at the sectoral level is not available for most countries. In this section we investigate the bias that may arise from mismeasuring factor income shares. For concreteness, let us focus on income shares of skilled labor. Suppose $\alpha_{skj} = \alpha_{skUS} + \nu_{jk}$. Then with some manipulations productivities can be written as[10]

$$E\left[log(\frac{A_{ijk}}{A_{iUSk}}|actual)\right] \approx E\left[log(\frac{A_{ijk}}{A_{iUSk}}|measured)\right] + E(\nu_{jk})log(\frac{w_{sj}}{w_{uj}}) + \tag{12}$$
$$E(\nu_{jk})(1-\alpha_{skUS}-\alpha_{capkUS}) + E[\nu_{jk}(\nu_{jk}-\alpha_{skUS}-\alpha_{capkUS})].$$

[10]To derive this, substitute the definition of skilled labor shares in the definition of "raw" productivity in the main text, divide by the value of the US, take logs, simplify and use $log(1+x) \approx x$.

Consequently, if the intensity differences are random, i.e., ν_{jk} is i.i.d. with $E(\nu_{jk}) = 0$ and $Var(\nu_{jk}) = \sigma_{jk}$, we get $E\left[log(\frac{A_{ijk}}{A_{iUSk}}|actual)\right] = E\left[log(\frac{A_{ijk}}{A_{iUSk}}|measured)\right] + \sigma_{jk}$. Hence, on average we tend to underestimate productivities in those sectors and countries that have very – but not systematically – different factor income shares than the US. Since this kind of measurement error is more likely to occur in poor countries, it may lead to underestimation of poor countries' productivities in specific sectors.

If poor countries have a systematically larger income share of skilled labor than the US, the more skill intensive the sector, we tend to predict systematically lower productivities of poor countries in skill intensive sectors. To see this, assume that in poor countries $E(\nu_{jk}) = \overset{(+)}{f}(\alpha_{sUS})$, a positive function of the skilled labor share in the US. Then the bias is negative, provided that the only negative term, $-(\alpha_{kUSs} + \alpha_{kUScap})E(\nu_{jk})$, does not dominate the other terms, which are all positive. It is unlikely, however, that poor countries have a systematically larger skilled labor income share in more skill intensive sectors than the US. If technological change is skill biased, the gap in the wage share of skilled labor between rich and poor countries is larger in more skill intensive sectors, so that we actually tend to overestimate the productivity of poor countries in skill intensive sectors. The intuition is that in this case we overestimate the cost of skilled labor inputs in poor countries in skill intensive sectors, which have on average higher skill premia than rich ones.

1.6. Comparing Estimates with Solow Residuals

To assess the validity of our method for computing sectoral TFPs we compare our productivity estimates with TFPs constructed from the OECD STAN database for the few countries and sectors where this is feasible. We assume sectoral production functions to be Cobb-Douglas with sectoral factor income shares equal to the ones of the US. For reasons of data availability, we are limited to 11 countries,[11] two factors – capital and efficient labor–, and eight sectors.[12]

We compute the Cobb-Douglas value added TFP index as

$$\frac{A_{jk}}{A_{USk}} \frac{p_{jk}}{p_{USk}} = \left(\frac{VA_{jk}}{VA_{USk}}\right) \left(\frac{K_{USk}}{K_{jk}}\right)^{\alpha_k} \left(\frac{H_{USk}}{H_{jk}}\right)^{1-\alpha_k} \tag{13}$$

[11] Austria, Belgium, Canada, Finland, France, Italy, Netherlands, Norway, Spain, United Kingdom, and United States.
[12] Those sectors are 31,32,...,38. Data is limited by the availability of information on gross fixed capital formation in STAN.

Note that we do not have information on sectoral price indices, so that our TFP measures are contaminated by relative prices, which may potentially severely bias these productivity indices.[13] To make our baseline productivities comparable with the ones computed from STAN, we aggregate trade data to fit the STAN definitions and construct wages for workers with no education.

In the last columns of Table 6 we present correlations and Spearman rank correlations between TFPs computed with our baseline specification and from the STAN database. The overall correlation between the two measures is 0.34 and the rank correlation is 0.3. These aggregate numbers hide a large variation in fit by sector. Rank correlation are quite high for sectors 37 (0.78) and 31 (0.55) but very low for other sectors.[14] Interestingly, the sectors with poor fit are those with high transport costs for which relative prices tend to vary much more across countries. Overall, the correlations are not overwhelming, but there clearly is a positive relation between the results of the two methods. One has to take into account that we have not only used a different approaches but also completely different data sets to compute the two sets of TFPs and that variation in relative prices may be severely distorting their comparability. In the end, the relative success of this robustness check together with the high correlation of our aggregate TFPs with the more reliable aggregate measures obtained using Hall and Jones' method makes us confident that we are indeed capturing productivity differences with our TFP measures constructed from trade data.

2. A Two Country General Equilibrium Model

In this section we present a two country general equilibrium version of the model used in the main text. Several features of the model in this section are more restrictive than the version estimated in the main text. These assumptions are just made to simplify the exposition and do not affect the basic results of the model.

There are two countries, Home and Foreign ($*$). Transport costs are allowed to be sector-specific and asymmetric and are denoted by τ_k and τ_k^*. We assume in this section that there are only two factors of production, capital, K and labor, L. The total number of varieties in each sector at the world level is

[13] Harrigan (1999) constructs international comparable sectoral price indices for some manufacturing sectors and finds large differences in sectoral prices even across a small number of OECD economies.

[14] Productitivities in sector 35 are not directly comparable, because we have removed some subsectors where exports depend mostly on the availability of oil resources from our dataset.

$N_k = n_k + n_k^*$.

It follows from the definition of the sectoral price index in the main text that the Home price index in sector k is defined as

$$P_k = \left[n_k p_k^{1-\epsilon_k} + n_k^*(p_k^* \tau^*)^{1-\epsilon_k}\right]^{\frac{1}{1-\epsilon_k}}. \tag{14}$$

A similar expression holds for the Foreign price index.

The revenue of a Home firm is given by the sum of domestic and Foreign revenue and using the expressions for Home and Foreign demand in the main text, we get

$$p_k q_{jk} = \sigma_k Y \left(\frac{p_k}{P_k}\right)^{1-\epsilon_k} + \sigma_k^* Y^* \left(\frac{p_k \tau_k}{P_k^*}\right)^{1-\epsilon_k}. \tag{15}$$

An analogous expression applies to Foreign Firms.

Given the demand structure firms optimally set prices as a fixed mark up over their marginal cost.

$$p_k = \frac{\epsilon_k}{\epsilon_k - 1} \frac{1}{A_{jk}} \left(\frac{w_j}{1-\alpha_k}\right)^{1-\alpha_k} \left(\frac{r_j}{\alpha_k}\right)^{\alpha_k} \tag{16}$$

Since firms can enter freely, in equilibrium they make zero profits and price at their average cost. Combining this with (16), it is easy to solve for equilibrium firm size, which depends positively on the fixed cost and the elasticity of substitution.

$$q_{jk} = q_k = f_k(\epsilon_k - 1) \tag{17}$$

Let us now solve for partial equilibrium in a single sector. For convenience, define the relative price of Home varieties in sector k, to be $\tilde{p}_k \equiv \frac{p_k}{p_k^*}$ and the relative fixed cost in sector k as $\tilde{f}_k \equiv \frac{f_k}{f_k^*}$. Dividing the Home market clearing condition by its Foreign counter part, one can derive an expression for $\frac{n_k}{n_k^*}$, the relative number of Home varieties in sector k.

A sector is not necessarily always located in both countries. In fact, if Home varieties are too expensive relative to Foreign ones, Home producers may not be able to recoup the fixed cost of production and do not enter this sector in Home. Consequently, if $\tilde{p} \geq \underline{p}_k$, we have that $n_k = 0$ and $n_k^* = \frac{\sigma_k(Y+Y^*)}{f_k^*(\epsilon_k - 1)}$, while if $\tilde{p} \leq \underline{p}_k$, the whole sector is located in Home, $n_k = \frac{\sigma_k(Y+Y^*)}{f_k(\epsilon_k - 1)}$ and $n_k^* = 0$.

For intermediate values of relative prices sectoral production is split across both countries, and the relative number of Home varieties is given by the following expression:

$$\frac{n_k}{n_k^*} = \frac{[\sigma_k Y(\tilde{p}_k \tilde{f}_k - \tilde{p}_k^{1-\epsilon_k}(\tau_k^*)^{\epsilon_k - 1}) + \sigma_k^* Y^*(\tilde{p}_k \tilde{f}_k - \tilde{p}_k^{1-\epsilon_k}\tau_k^{1-\epsilon_k})]}{[\sigma_k^* Y^* \tilde{p}_k^{1-\epsilon_k}(\tau_k^*)^{\epsilon_k - 1}(\tilde{p}_k \tau_k^{1-\epsilon_k} - \tilde{p}_k \tilde{f}_k) - \sigma_k Y \tilde{p}_k^{1-\epsilon_k}\tau_k^{1-\epsilon_k}(\tilde{p}_k \tilde{f}_k - \tilde{p}_k^{1-\epsilon_k}(\tau_k^*)^{\epsilon_k - 1})]}, \tag{18}$$

for $\tilde{p}_k \in (\underline{p}_k, \bar{p}_k)$, where

$$\underline{p}_k = \left[\frac{(\sigma_k^* Y^* + \sigma_k Y)(\tau_k^*)^{\epsilon_k - 1}\tau_k^{1-\epsilon_k}}{\sigma_k Y \tau_k^{1-\epsilon_k}\tilde{f}_k + \sigma_k^* Y^* (\tau_k^*)^{\epsilon_k - 1}\tilde{f}_k} \right]^{1/\epsilon_k} \tag{19}$$

and

$$\bar{p}_k = \left[\frac{\sigma_k^* Y^* \tau^{1-\epsilon_k} + \sigma_k Y (\tau_k^*)^{\epsilon_k - 1}}{\tilde{f}_k \sigma_k^* Y^* + \tilde{f}_k \sigma_k Y} \right]^{1/\epsilon_k}. \tag{20}$$

Defining the Home revenue share in industry k as $v_k \equiv \frac{n_k p_k x_k^s}{n_k p_k x_k^s + n_k^* p_k^* x_k^{s*}}$ we can derive that $v_k = 0$ if $\tilde{p}_k \geq \bar{p}_k$. On the other hand, v_k is given by $\frac{1}{1+(\frac{n}{n^*})^{-1}\tilde{p}^{-1}\tilde{f}^{-1}}$ if $\tilde{p}_k \in (\underline{p}_k, \bar{p}_k)$ and finally $v_k = 1$ if $\tilde{p}_k \leq \underline{p}_k$.

The model is closed by substituting the pricing condition (16) into $\tilde{p}$ and the expressions for v_k in the factor market clearing conditions for Home and Foreign.

$$\sum_{k=1}^{K}(1-\alpha_k)v_k\sigma_k(Y+Y^*) + (1-\alpha_{NT})\sigma_{NT}Y = wL \tag{21}$$

$$\sum_{k=1}^{K}\alpha_k v(k)\sigma_k(Y+Y^*) + \alpha_{NT}\sigma_{NT}Y = rK \tag{22}$$

$$\sum_{k=1}^{K}(1-\alpha_k)(1-v_k)\sigma_k(Y+Y^*) + (1-\alpha_{NT})\sigma_{NT}Y^* = w^*L^* \tag{23}$$

$$\sum_{k=1}^{K}\alpha_k(1-v_k)\sigma_k(Y+Y^*) + \alpha_{NT}\sigma_{NT}Y^* = r^*K^* \tag{24}$$

Here σ_{NT} is the share of expenditure spent on non-tradable goods. Normalizing one relative factor price, we can use 3 factor market clearing conditions to solve for the remaining factor prices.

One can show that the Home revenue share in sector k, v_k, is decreasing in the relative price of Home varieties $\tilde{p}_k$. This implies that countries have larger revenue shares in sectors in which they can produce relatively cheaply. Cost advantages may arise both because a sector uses the relatively cheap factor intensively and because of high relative sectoral productivity.

2.1. Romalis' Model

In the special case in which sectoral productivity differences are absent, $\frac{A_k}{A_k^*} = 1$ for all $k \in K$, relative fixed costs of production are equal to one, $\tilde{f}_k = 1 \ \forall \ k \in K$, sectoral elasticities of substitution are the same in all sectors, $\epsilon_k = \epsilon$, trade costs are symmetric and identical across sectors $\tau_k = \tau_k^* = \tau$ and preferences are identical, $\sigma_k = \sigma_k^*$, the model reduces to Romalis (2004) model.

In his framework, the relative price of Home varieties, $\tilde{p}_k = \frac{\left(\frac{w}{1-\alpha_k}\right)^{1-\alpha_k}\left(\frac{r}{\alpha_k}\right)^{\alpha_k}}{\left(\frac{w^*}{1-\alpha_k}\right)^{1-\alpha_k}\left(\frac{r^*}{\alpha_k}\right)^{\alpha_k}}$, is decreasing in the capital intensity, α_k, if and only if Home is relatively abundant in capital, i.e., $\frac{K}{L} > \frac{K^*}{L^*}$.

Factor prices are not equalized across countries because of transport costs, which gives Home a cost advantage in the sectors that use its abundant factor intensively. This in turn leads to a larger market share of the Home country in those sectors as consumers shift their expenditure towards the relatively cheap Home varieties. This is the intuition for the Quasi-Heckscher-Ohlin prediction that countries are net exporters of those goods which use their relatively abundant factor intensively. The main advantage of this model is that it solves the production indeterminacy present in the standard Heckscher-Ohlin model with more goods than factors whenever countries are not fully specialized and that it provides a direct link between factor abundance and sectoral trade patterns. This makes it ideal for empirical applications.

2.2. A Ricardian Model

If we make the alternative assumption that all sectors use labor as the only input, i.e., $\alpha_k = 0$ for all $k \in K$ and we order sectors according to Home comparative advantage, such that $\frac{A_k}{A_k^*}$ is increasing in k, we obtain a Ricardian model. The advantage of this model is that because of love for variety, consumers are willing to buy both Home and Foreign varieties in a sector even when they do not have the same price. The setup implies that $\tilde{p}_k = \frac{w}{w^*}\frac{A_k^*}{A_k}$ is decreasing in k, so that Home offers lower relative prices in sectors with higher

k. Consequently, Home captures larger market shares in sectors with larger comparative advantage since v_k is decreasing in $\tilde{p}_k$ and $\tilde{p}_k$ is decreasing in $\frac{A_k}{A_k^*}$.

2.3. The Hybrid Ricardo-Heckscher-Ohlin Model

In the more general case comparative advantage is both due to differences in factor endowments and due to differences in sectoral productivities. Note that $\tilde{p}_k$ is given by the following expression:

$$\tilde{p}_k = \frac{\frac{1}{A_k}\left(\frac{w}{1-\alpha_k}\right)^{1-\alpha_k}\left(\frac{r}{\alpha_k}\right)^{\alpha_k}}{\frac{1}{A_k^*}\left(\frac{w^*}{1-\alpha_k}\right)^{1-\alpha_k}\left(\frac{r^*}{\alpha_k}\right)^{\alpha_k}} \tag{25}$$

Assume again that Home is relatively capital abundant, $\frac{K}{L} > \frac{K^*}{L^*}$. Then, conditional on $\frac{w}{r}, \frac{w^*}{r^*}$, Home has lower prices and a larger market share in sectors where $\frac{A_k}{A_k^*}$ is larger. In addition, factor prices depend negatively on endowments unless the productivity advantages are systematically much larger in sectors that use the abundant factor intensively. A very high relative productivity in the capital intensive sectors can increase demand for capital so much that $\frac{w}{r} < \frac{w^*}{r^*}$ even though $\frac{K}{L} > \frac{K^*}{L^*}$. As long as this is not the case, locally abundant factors are relatively cheap and – holding constant productivity differences – this increases market shares in sectors that use the abundant factor intensively.

The model is illustrated in Figure 1. In this example, $\epsilon_k = 4$, Home is relatively capital abundant, $\frac{K/L}{K^*/L^*} = 4$, and transport costs are high, $\tau_k = \tau_k^* = 2$. The panels of Figure 1 plot Homes' relative productivity, Homes' sectoral revenue share, Homes' relative prices, as well as Homes' net exports, Homes' exports relative to production and Homes' imports relative to production against the capital intensity of the sectors, which is ordered on the zero-one interval. In the first case (solid lines) there are no productivity differences between Home and Foreign. Because Home is capital abundant it has lower rentals and higher wages which leads to lower prices and larger revenue shares in capital intensive sectors. In addition, Home is a net importer in labor intensive sectors and a net exporter in capital intensive ones and its exports relative to production are larger in capital intensive sectors, while its imports relative to production are much larger in labor intensive sectors. This illustrates neatly the Quasi-Heckscher-Ohlin prediction of the model.

In the second case (dashed lines) – besides being more capital abundant – Home also has systematically higher productivities in more capital intensive sectors. This increases Home's comparative advantage in capital intensive sectors even further. The consequence of higher productivity is an increased demand for both factors that increases home factor prices and makes Home even less competitive in labor abundant sectors, while the relative price in capital abundant sectors is lower than without productivity differences. The result is a higher revenue share in capital intensive sectors and more extreme import and export patterns than without productivity differences.

Figure 2 is an example of the Quasi-Rybczynski effect. Initially both Home and Foreign have the same endowments, $\frac{K/L}{K^*/L^*} = 1$, and Home has a systematically higher productivity than Foreign in capital intensive sectors (solid lines), which explains Homes' larger market share in those sectors. In the case with the dashed lines Home has doubled its capital stock, so that now $\frac{K/L}{K^*/L^*} = 2$. This leads to an expansion of production and revenue shares in the capital intensive sectors and a decline of production in the labor intensive sectors. The additional capital is absorbed both through more capital intensive production and an expansion of production in capital intensive sectors. The increased demand for labor in those sectors drives up wages and makes Home less competitive in labor intensive sectors.

Summing up, the general prediction of the Hybrid-Ricardo-Heckscher-Ohlin model is that exporting countries capture larger market shares in sectors in which their abundant factors are used intensively (Quasi-Heckscher-Ohlin prediction) and in which they have high productivities relative to the rest of the world (Quasi-Ricardian prediction). In addition, the model has a Quasi-Rybczynski effect. Holding productivities constant, factor accumulation leads to an increase in revenue shares in sectors that use the factor intensively and a decrease in those sectors that use the factor little.

References

Chor, D. (2010). 'Unpacking sources of comparative advantage: a quantitative approach', *Journal of International Economics,* forthcoming.

Eaton, J. and Kortum, S. (2002). 'Technology, geography, and trade', *Econometrica*, vol. 70(5), pp. 1741–1779.

Ethier, W. (1982). 'National and international returns to scale in the modern theory of international trade', *American Economic Review*, vol. 72, pp. 389–405.

Fadinger, H. and Fleiss, P. (2008). 'Trade and sectoral productivity', Working Paper 2008-05, ECARES.

Finicelli, A., Pagano, P., and Sbracia, M. (2008). 'Trade-revealed TFP', Working Paper 729, Banca d'Italia.

Harrigan, J. (1999). 'Estimation of cross-country differences in industry production functions', *Journal of International Economics*, vol. 47(2), pp. 267–293.

Helpman, E., Melitz, M., and Rubinstein, Y. (2008). 'Estimating trade flows: trading partners and trading volumes', *Quarterly Journal of Economics*, vol. 123(2), pp. 441–487.

Jones, C. I. (2010). 'Intermediate goods and weak links in the theory of economic development', *American Economic Journal: Macroeconomics,* forthcoming.

Melitz, M. (2003). 'The impact of trade on intra-industry reallocations and aggregate industry productivity', *Econometrica*, vol. 71(6), pp. 1695–1725.

Melitz, M. and Ottaviano, G. I. P. (2008). 'Market size, trade and productivity', *Review of Economic Studies*, vol. 75, pp. 295–316.

Rivera-Batiz, L. A. and Romer, P. (1991). 'Economic integration and economic growth', *Quarterly Journal of Economics*, vol. 106, pp. 531–555.

Romalis, J. (2004). 'Factor proportions and the structure of commodity trade', *American Economic Review*, vol. 94(1), pp. 67–97.

Sauré, P. (2009). 'Bounded love for variety and patterns of trade', Working Paper 2009-10, Swiss National Bank.

Table 1: Robustness of TFP estimates

		Hausman-Taylor		Number of Firms		Heckman		Heterogeneous Firms		Eaton and Kortum		Solow Residual		
isic	Sector Name	Correl	Spearman	Correl	Spearman	Correl	Spearman	Correl	Spearman	Correl	Spearman	isic	Correl	Spearman
311	Food	0.99	0.99	0.93	0.94	0.94	0.95	0.68	0.78	0.92	0.92	31	0.55	0.49
313	Beverages	0.97	0.96	0.91	0.95	0.91	0.94	0.81	0.85	0.91	0.89			
321	Textiles	0.94	0.95	0.91	0.93	0.93	0.92	0.63	0.83	0.93	0.96	32	0.33	0.24
322	Apparel	0.99	0.99	0.70	0.78	0.71	0.72	0.61	0.68	0.85	0.85			
323	Leather	0.98	0.99	0.71	0.83	0.79	0.81	0.78	0.82	0.82	0.86			
324	Footwear	0.96	0.97	0.89	0.92	0.90	0.89	0.58	0.78	0.85	0.90			
331	Wood	0.85	0.88	0.95	0.96	0.97	0.96	0.61	0.75	0.96	0.96	33	0.40	0.12
332	Furniture	0.98	0.97	0.52	0.76	0.68	0.73	0.73	0.72	0.72	0.64			
341	Paper	0.90	0.92	0.94	0.97	0.92	0.97	0.70	0.78	0.97	0.96	34	0.44	0.38
342	Printing	0.97	0.97	0.75	0.90	0.88	0.86	0.80	0.83	0.92	0.91			
351	Chemicals	0.97	0.97	0.93	0.94	0.93	0.92	0.57	0.66	0.93	0.95	35	0.13	0.09
352	Other Chemic	0.94	0.94	0.95	0.97	0.94	0.97	0.70	0.79	0.95	0.96			
355	Rubber	0.90	0.91	0.89	0.93	0.93	0.94	0.74	0.82	0.96	0.96			
356	Plastic	0.98	0.98	0.84	0.94	0.86	0.94	0.83	0.95	0.85	0.8			
361	Pottery	0.97	0.98	0.36	0.63	0.19	0.54	0.25	0.54	0.79	0.72	36	0.15	0.19
362	Glass	0.96	0.97	0.83	0.88	0.82	0.85	0.68	0.76	0.96	0.95			
369	Other Non-Metal	0.91	0.94	0.96	0.95	0.95	0.95	0.74	0.83	0.98	0.98			
371	Iron and Steel	0.83	0.89	0.97	0.98	0.97	0.98	0.65	0.78	0.96	0.97	37	0.78	0.73
372	Non-Ferrous	0.85	0.88	0.97	0.97	0.98	0.98	0.67	0.74	0.95	0.94			
381	Fabricated Metal	0.95	0.96	0.85	0.89	0.85	0.86	0.76	0.81	0.96	0.95	38	0.43	0.26
382	Machinery	0.93	0.95	0.88	0.93	0.85	0.92	0.73	0.84	0.96	0.95			
383	Electrical Machinery	0.94	0.96	0.81	0.91	0.80	0.90	0.76	0.84	0.96	0.94			
384	Transport	0.86	0.91	0.85	0.91	0.84	0.90	0.56	0.70	0.92	0.93			
385	Scientific	0.96	0.97	0.35	0.78	0.39	0.79	0.40	0.80	0.86	0.81			

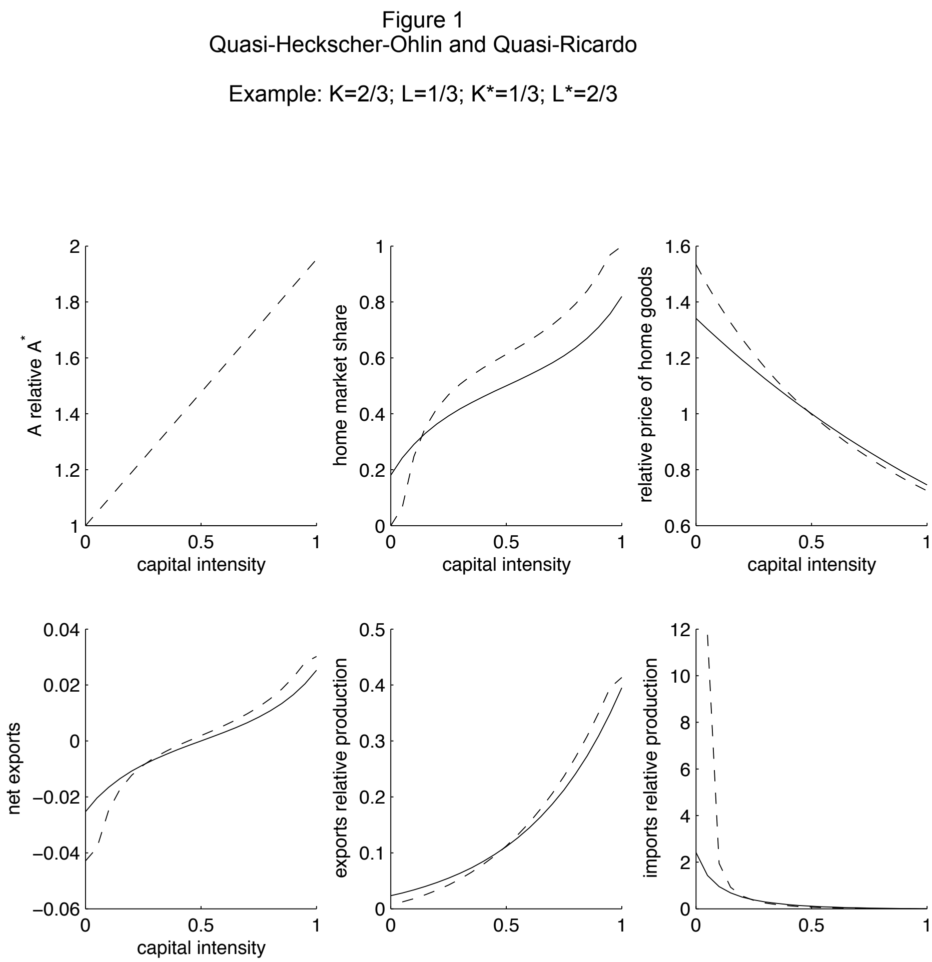

Figure 1: Quasi-Heckscher-Ohlin and Quasi-Ricardo effects

Figure 2
Quasi-Rybczynski Effect

Example: K=1/3; L=1/3; K*=1/3; L*=1/3; Home doubles Capital stock K'=2/3

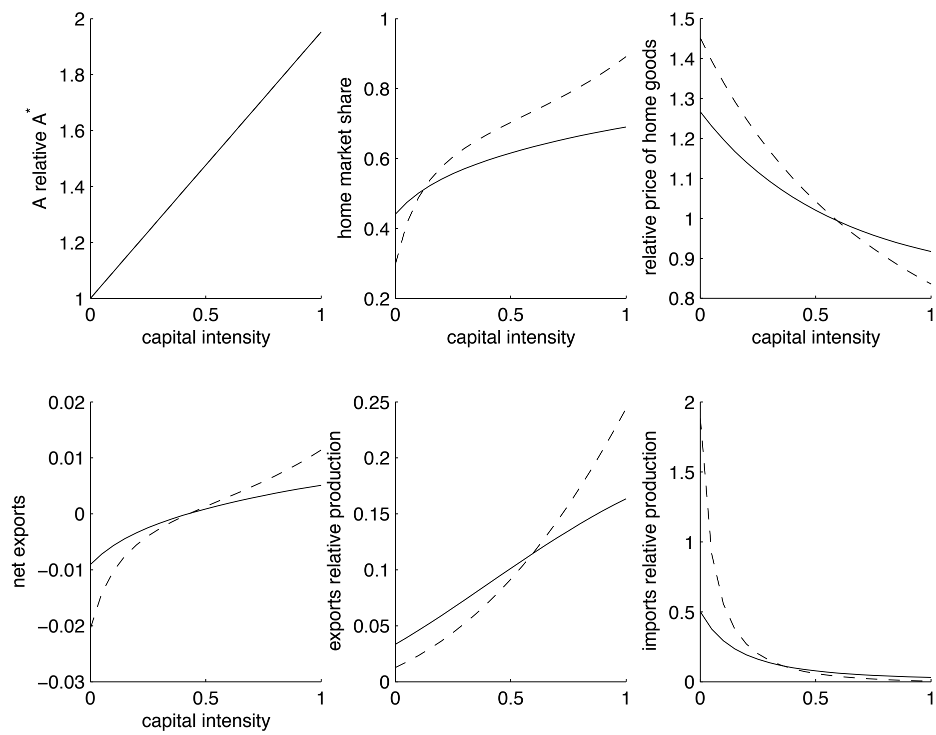

Figure 2: Quasi-Rybczynski effect